A NEO-PHENOMENALISTIC APPROACH TO RELATIVITY & PHYSICS AT THE QUANTUM LEVEL

The following is a talk delivered in a Saturday Seminar on the subject of *Phenomenalism* at the University of West of England (UWE) on the 10th September 2005. The Seminar was organised by Dr. Michael C. Duffy, of Sunderland University and the talk was by Dr. Anthony D. Osborne, of the Department of Mathematics, Keele University, UK. The subject of the talk was that of Neo-Phenomenalism, in the *Normal Realist* tradition of science-philosophy established in the 1970s by N. Vivian Pope, of Swansea.

Since the early 1980s, Pope and Osborne have been collaborators in an ongoing Philosophy-Mathematics project at Keele University. This project is known by its acronym, POAMS (the Pope-Osborne Angular Momentum Synthesis). The development of the philosophy is shown elsewhere, (see, *e.g.* 'Normal Realism' on Google.) In this present paper, Dr. Osborne concentrates on the mathematical details of the Philosophical Synthesis.

A NEO-PHENOMENALISTIC APPROACH TO RELATIVITY & PHYSICS AT THE QUANTUM LEVEL

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The Normal Realist Approach to Special Relativity

It is postulated that

(A1) The laws of physics are the same in all inertial frames.
(A2) Observational distance and time have a constant ratio of units, c, for all observers sharing that same conventional choice of units.

Consider a clock X moving with uniform velocity, *i.e.* with constant speed v along a rectilinear path, relative to an observer O. After passage of time t recorded by O, X moves a distance $r = vt$. It follows, by using the conversion factor c, X moves a distance-time r/c. Let us suppose that the same time interval as recorded by X's clock is τ, as observed by O. It then follows simply by applying Pythagoras's theorem in the associated two-dimensional time diagram with coordinates r/c and τ that

$$t^2 = \tau^2 + (r/c)^2 \qquad (1)$$

Since $r = vt$ it follows from (1) that

$$\tau = (1 - (v^2/c^2))^{1/2}t \qquad (2)$$

- the standard time dilation formula of SR.

Consider any clock X which moves with uniform velocity relative to an observer O. Then relative to O's spatial Cartesian coordinates x, y and z, after a passage of time t relative to O, X travels a distance r where $r^2 = x^2 + y^2 + z^2$, so that by (1),

$$-c^2\tau^2 = -c^2t^2 + x^2 + y^2 + z^2 \qquad (3)$$

The right-hand-side of (3) immediately provides and invariant quantity for all inertial observers, the **separation** between two events, the basis for Minkowski space-time (MST).

Now suppose clock X moves with non-constant velocity $\mathbf{v}(t)$ relative to observer O and let $v(t) = \|\mathbf{v}(t)\|$ denote the speed of the clock relative to O. It follows by (2) that in this case, the relatively moving clock X records proper time τ given by

$$\tau = \int \sqrt{(1 - v^2(t)/c^2)}\, \mathrm{d}t \qquad (4)$$

Relative to Cartesian coordinates (x, y, z), the speed of a moving object satisfies

$$v^2(t) = \|\mathbf{v}(t)\|^2 = (\mathrm{d}x/\mathrm{d}t)^2 + (\mathrm{d}y/\mathrm{d}t)^2 + (\mathrm{d}z/\mathrm{d}t)^2$$

Then (4) gives

$$c^2(\mathrm{d}\tau/\mathrm{d}t)^2 = c^2 - v^2(t)$$
$$= c^2 - (\mathrm{d}x/\mathrm{d}t)^2 - (\mathrm{d}y/\mathrm{d}t)^2 - (\mathrm{d}z/\mathrm{d}t)^2$$

$$\Longrightarrow -c^2\mathrm{d}\tau^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

In this way, the time dilation formula directly provides the *metric* for MST.

It follows directly from (4) that

$$\mathrm{d}t/\mathrm{d}\tau = (1 - v^2(t)/c^2)^{-1/2} = \gamma(t)$$

This result is crucial in relativistic dynamics. This formula, together with considerations of conservation of momentum, leads to the mass increase with velocity formula and to the equivalence of mass and energy.

Consider two observers O and O', each situated at the origin of a set of Cartesian axes labelled x, y and z, and x', y' and z' respectively. Suppose that O' moves along the x-axis in the positive direction with constant speed v relative to O such that the x', y' and z' axes remain parallel to the x, y and z axes respectively. Let t be some passage of time as recorded by a clock in O's inertial frame and let t' be the corresponding passage of time as recorded by a clock in O''s inertial frame. We suppose that the observers O and O' synchronise clocks when they coincide at $t = 0$, so that when $t = 0$ at (0, 0, 0) relative to O, $t' = 0$ at (0, 0, 0) relative to O'. It follows from (A1) exactly as in SR,

$$x' = \gamma(x - vt) \tag{5}$$

for some constant γ to be determined. It then follows that since O travels at speed v in the negative x' direction relative to O', then

$$x = \gamma(x' + vt')$$

$$\Longrightarrow x = \gamma(\gamma x - \gamma vt + vt')$$

$$\Longrightarrow t' = \gamma(t - (1 - 1/\gamma^2)x/v) \tag{6}$$

After time t relative to O, O' has moved a distance $x = vt$. Then the same passage of time, t', as recorded by O''s clock relative to O is given by (6) with $x = vt$, so that

$$t' = \gamma(t - (1 - 1/\gamma^2)t) = t/\gamma \tag{7}$$

But here, $t' = \tau$, the proper time recorded by the travelling clock, so that (7) is the same as (2), immediately giving

$$\gamma = (1 - v^2/c^2)^{-1/2}$$

from which the special Lorentz transformation, consisting of (5) and (6) follows.

The Pope-Osborne Angular Momentum Synthesis

It is postulated that all *free* motion is naturally orbital and that angular momentum is holistically conserved. Such natural orbital motion is explained in terms of conservation of momentum, without the need to postulate any *in vacuo* 'gravitational force' being responsible.

The *natural orbit* of any particle is the path described by the particle when all restrictions on its motion are removed, that is, when the particle moves freely under the influence of nothing but its own angular momentum.

Consider an isolated paired system, in which a particle P of mass m orbits a larger body B of mass M, where $M \gg m$. P can be considered as orbiting an origin O, which is the centre of mass of B.

Orbital angular momentum, $\mathbf{L}$, of P, about O:

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v},$$

where $\mathbf{r}$ is the position vector of P and $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ is the velocity of P, relative to O, at any instant of time t. Note $\mathbf{L}$ is perpendicular to the plane containing $\mathbf{r}$ and $\mathbf{v}$.

By hypothesis, $\mathbf{L}$ is constant in time. Hence, the orbit of P lies in a plane and $L = \|\mathbf{L}\|$ is a constant. Also, since $\mathrm{d}\mathbf{L}/\mathrm{d}t = \mathbf{0}$,

$$\mathrm{d}\mathbf{L}/\mathrm{d}t = \mathbf{v} \times m\mathbf{v} + \mathbf{r} \times m\mathbf{a} = \mathbf{r} \times m\mathbf{a} = \mathbf{0},$$

where $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ is the acceleration of P relative to O. Since $m \neq 0$, $\mathbf{r} \neq \mathbf{0}$ and $\mathbf{a} \neq \mathbf{0}$, this implies that $\mathbf{a}$ must be parallel to $\mathbf{r}$ and so the acceleration of P is directed towards O.

Since the orbit of P lies in a plane, we use plane polar coordinates r and θ, where $r = \|\mathbf{r}\|$ and θ is the angle between the radial vector $\mathbf{r}$ and some fixed radial axis.

Exactly as in Newtonian theory, it then follows that the *orbital speed*, $v = \|\mathbf{v}\|$, of P is given by

$$v^2 = (\mathrm{d}r/\mathrm{d}t)^2 + r^2(\mathrm{d}\theta/\mathrm{d}t)^2 \qquad (8)$$

Also,

$$a = \|\mathbf{a}\| = |\ddot{r} - r\dot{\theta}^2| \ (9) \quad \text{and} \quad L = mr^2\dot{\theta} \ (10)$$

Hence, if P's orbit is a *circle* of radius r and centre O, then

$$L = mvr \qquad (11)$$

In general, since the acceleration of P is directed towards O, it must take the form

$$\mathbf{a} = d^2\mathbf{r}/dt^2 = -(h(r)/r)\mathbf{r} \quad (12)$$

thus providing the general equation of motion for the orbit of *P*. Then (9), (10) and (12) give

$$a = |\ddot{r} - r(\dot{\theta})^2| = |\ddot{r} - L^2/(m^2r^3)| = h(r). \quad (13)$$

For *initial simplicity*, we require the orbit of *P* about *O* to be *closed*. It then follows by Bertrand's theorem that either $h(r) = \alpha/r^2$ or $h(r) = \alpha r$, for some positive constant α. The only choice of $h(r)$ which agrees with observational and empirical evidence is $h(r) = \alpha/r^2$. Let $\alpha = GM$, where *G* is simply a new constant, so that (10) and (13) are then the same as the corresponding Newtonian equations.
In particular, (13) gives

$$d^2r/dt^2 - L^2/(m^2r^3) = -GM/r^2 \quad (14)$$

It is thus possible for *P* to follow a natural circular orbit, which, using (11) is given by

$$v^2 = GM/r \quad (15)$$

As in Newtonian theory, (14) may be integrated to give

$$(dr/dt)^2 + L^2/(m^2r^2) = 2GM/r + K \quad (16)$$

where *K* is a constant. Since *P*'s orbit is assumed to be closed, $K < 0$.

Multiplying (16) by $(mr^2/L)^2$, using (10) and then letting $u = 1/r$ gives, as in Newtonian theory,

$$(du/d\theta)^2 + u^2 = (2GMm^2/L^2)u + Km^2/L^2 \quad (17)$$

The solutions of (17) which are closed curves are ellipses.

Orbital Time Dilation

Consider again a freely moving particle *P* orbiting a body *B* of mass *M*. It follows by (15) and (4) that if the orbit of *P* is *circular*, then since the speed of *P* is constant, the time dilation determined by velocity effects alone is

$$\tau = \sqrt{(1 - GM/(rc^2))}t \quad (18)$$

Here, τ is the proper time as recorded by *P*'s clock and *t* is the same passage of time as recorded by an observer for whom the speed of *P* is given by (15). The time *t* is usually called 'coordinate time' in Relativity. This is what POAMS calls *deep space time* (DST).

If *P* follows a closed, non-circular natural orbit about *B*, it follows by (4), (8), (16) and (10) that the time dilation caused by velocity alone is, in this case, given by

$$\tau = \int\sqrt{(1 - 2GM/(rc^2) - K/c^2)}\, dt$$

However, these results would apply equally well if *P* were travelling in a straight line with the same speed *v*.

Hence, these equations do not take into account the fact that the natural orbits in POAMS are not straight lines. In fact, they are equivalent to (4), which implies the metric for MST, in which freely moving objects follow straight lines.

If *P* is freely moving in a *circular* orbit about *B*, it then follows that the proper time, τ, as recorded by a clock travelling with *P*, relative to DST *t*, taking only velocity effects into account, is given by (18). Suppose the body *B* is spherically symmetric and static. Then any additional time dilation effect can depend only on *M* and the distance, *r*, of *P* from *B*. We suppose that any such additional effect is proportional to M/r and so postulate that the true proper time as recorded by *P*'s clock relative to DST is given by

$$\tau = \sqrt{(1- GM/(rc^2) - aGM/(rc^2))}\,t \qquad (19)$$

where *a* is a constant to be determined. Let $\mathcal{M} = GM/c^2$. Then by (15), (19) becomes

$$\tau = \sqrt{(1 - v^2/c^2 - a\mathcal{M}/r)}\,t$$
$$\Longrightarrow -c^2(\mathrm{d}\tau/\mathrm{d}t)^2 = -c^2 + v^2 + a\mathcal{M}c^2/r$$

Since the speed of *P* in its circular orbit satisfies $v = r(\mathrm{d}\theta/\mathrm{d}t)$ by (15), it follows that

$$-c^2\mathrm{d}\tau^2 = -c^2(1 - a\mathcal{M}/r)\mathrm{d}t^2 + r^2\mathrm{d}\theta^2 \qquad (20)$$

Equation (20) may be treated as a special case of the metric

$$-c^2\mathrm{d}\tau^2 = -c^2(1 - a\mathcal{M}/r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 \qquad (21)$$

for a three-dimensional space-time when *r* is a constant. The coefficient of dr^2, *i.e.* *A*, in (21) must be a function of *r* alone, since the central mass *M* is assumed to be spherically symmetric and static. In MST, the paths of freely moving particles are *geodesics*. In order for circular orbits satisfying (15) to be natural orbits in space, we require that these orbits are geodesics in the space-time with metric (21). It follows by standard differential geometry that all geodesics in this structure must satisfy

$$\mathrm{d}t/\mathrm{d}\tau = (\alpha/c^2)(1 - a\mathcal{M}/r)^{-1} \qquad (22a)$$

$$\mathrm{d}\theta/\mathrm{d}\tau = \beta/r^2 \qquad (22b)$$

$$2\mathrm{d}^2r/\mathrm{d}\tau^2 + (ac^2\mathcal{M}/Ar^2)(\mathrm{d}t/\mathrm{d}\tau)^2 + (A'(r)/A)(\mathrm{d}r/\mathrm{d}\tau)^2 = (2r/A)(\mathrm{d}\theta/\mathrm{d}\tau)^2 \qquad (22c)$$

where α and β are constants. In the special case when r is a constant, (22c) reduces to

$$(ac^2\mathcal{M}/2r^3)(\mathrm{d}t/\mathrm{d}\tau)^2 = (\mathrm{d}\theta/\mathrm{d}\tau)^2$$

$$\Longrightarrow (\mathrm{d}\theta/\mathrm{d}t)^2 = ac^2\mathcal{M}/2r^3$$

It then follows that for any circular orbit, since $v = r(\mathrm{d}\theta/\mathrm{d}t)$,

$$v^2 = ac^2\mathcal{M}/2r = aGM/2r$$

Hence, in order for circular orbits given by (15) to be geodesics, $\underline{a = 2}$. Then by (19), the true proper time recorded by P's clock in its circular orbit relative to DST is given, as in GR, by

$$\tau = \sqrt{(1 - 3GM/(rc^2))}\,t$$

Schwarzschild Space-time as a Consequence of Time Dilation

The coefficient $A(r)$ in the metric (21) with $a = 2$, which is essentially the time dilation formula for any natural orbit in POAMS, can be determined by the fact that the geodesics in this space-time must give rise to curves in space that are ellipses, or at least almost ellipses, as dictated by Newtonian theory, POAMS and observation.

The geodesics for the manifold with metric (21) are given by equations (22) with $a = 2$. It also follows from the metric (21) that along any geodesic,

$$c^2(1 - 2\mathcal{M}/r)(\mathrm{d}t/\mathrm{d}\tau)^2(\mathrm{d}\tau/\mathrm{d}\theta)^2 - A(r)(\mathrm{d}r/\mathrm{d}\theta)^2 - r^2 = c^2(\mathrm{d}\tau/\mathrm{d}\theta)^2$$

Then from equations (22a) and (22b) it follows that

$$(\alpha^2r^4/\beta^2c^2) - A(r)(1 - 2\mathcal{M}/r)(\mathrm{d}r/\mathrm{d}\theta)^2 - r^2(1 - 2\mathcal{M}/r) = (c^2r^4/\beta^2)(1 - 2\mathcal{M}/r)$$

Letting $r = 1/u$ then gives

$$A(u)(1 - 2\mathcal{M}u)(\mathrm{d}u/\mathrm{d}\theta)^2 + u^2 = K^* + 2\mathcal{M}uc^2/\beta^2 + 2\mathcal{M}u^3 \qquad (23)$$

where $K^* = (\alpha^2/c^2 - c^2)/\beta^2$. Equation (23) describes any non-circular natural orbit in POAMS. The term $2\mathcal{M}u^3$ in (23) is almost negligible for planetary orbits or satellites orbiting planets. Hence, letting $K^* = Km^2/L^2$ and $\beta^2 = L^2/m^2$, (23) is very nearly

$$A(u)(1 - 2\mathcal{M}u)(\mathrm{d}u/\mathrm{d}\theta)^2 + u^2 = (2GMm^2/L^2)u + Km^2/L^2$$

This equation reduces to (17) and provides elliptical orbits only if $A(u) = (1 - 2\mathcal{M}u)^{-1}$ and hence the metric (21) must read

$$-c^2d\tau^2 = -c^2(1 - 2\mathcal{M}/r)dt^2 + (1 - 2\mathcal{M}/r)^{-1}dr^2 + r^2d\theta^2 \qquad (24)$$

This is the metric for the 'equatorial plane' of Schwarzschild space-time as derived in GR. It is this metric which predicts the perihelion shift effect.

Hence, in POAMS, conservation of angular momentum together with consideration of time dilation effects inevitably leads to the perihelion shift phenomenon.

Notice that since (24) is derived from (19) with $a = 2$, it follows that $r > 3\mathcal{M}$ in (24). Hence, in contrast to GR, in POAMS it is not possible to extrapolate and apply this metric to 'gravitational collapse' to obtain pathological 'space-time singularities'.

For initial simplicity, it was assumed that in any isolated two-body system, orbits are closed. Then Bertrand's theorem gives (14) as the equation of motion and, as a consequence, geodesic orbits are ellipses. However, once time dilation effects are taken into consideration, POAMS predicts, just as in GR, that the natural orbits are given by (23) with $A(u) = (1 - 2\mathcal{M}u)^{-1}$.

Spin in POAMS

In POAMS, any effects of spin are instantaneously correlated with the total angular momentum. POAMS takes account of this by postulating that the orbital kinetic energy of an orbiting spinning particle depends on both its spin kinetic energy and the orbital kinetic energy it would have if it were not spinning. This new approach implies that the 'gravitational constant' G, has to be replaced by a more general function $\mathcal{G}$ of the orbital parameters.

Once again, only the case of a particle P, of mass m, orbiting a body B, of mass M, will be considered. Suppose for the moment that P does not spin and follows a natural elliptical orbit about B, with radial coordinate $r_0(t)$, and with *angular speed* $\omega_0(t)$. P's orbital kinetic energy, K_o, is given by

$$K_o = mv_0^2/2 \qquad (25)$$

where $v_0(t)$ is the orbital speed of P in its natural orbit. It follows by (8) and (10) that

$$K_o = m(r_0^{\bullet})^2/2 + L\omega_0/2$$

$$\Longrightarrow\ L = 2K_o/\omega_0 - m(r_0^{\bullet})^2/\omega_0 \qquad (26)$$

Now let P spin on its axis while orbiting the body B. POAMS postulates that the orbital parameters of P's orbit will be directly affected by P's spin. Hence, by (26), when spinning, P follows a new natural orbit given by

$$L = 2\mathcal{K}/\omega_s - m(r_s^{\bullet})^2/\omega_s \qquad (27)$$

where now $\mathcal{K}$ is P's orbital kinetic energy, and $r_s(t)$ and $\omega_s(t)$ are the parameters of P's natural orbit, under the influence of its spin. POAMS postulates that $\mathcal{K}$ depends on both the orbital kinetic energy, K_o, of P of the natural orbit it would follow if it were not spinning and its spin kinetic energy, K_s.

Clearly, the maximum possible effect due to spin occurs when P either spins in the same direction as its orbital motion, so that K_s acts with K_o, or in the opposite direction to its orbital motion, in which case K_s acts against K_o. In the first case we postulate that

$$\mathcal{K} = K_o + K_s$$

and in the second case, we postulate that

$$\mathcal{K} = |K_o - K_s|.$$

Once $\mathcal{K}$ has been determined, it follows by (27) and (10) that

$$L^2 = 2\mathcal{K}mr_s^2 - m^2(\dot{r}_s)^2 r_s^2.$$

Since L can also be determined, $r_s(t)$ can be determined from this equation and then $\omega_s(t)$ can be found using (10). The fact that P's orbital parameters are affected by its spin gives rise to the fact that the equation of motion, (14), must be adapted to read

$$\ddot{r}_s - r_s\omega_s^2 = -\mathcal{G}M/r_s^2$$

that is, G is replaced by a function $\mathcal{G}$ which depends on the orbital parameters r_s and ω_s. $\mathcal{G}$ reduces to G when and only when (26) describes any natural, spin-less orbit of P.

If the orbit of P is *circular*, this process is considerably simplified. In this case

$$v_s^2 = 2\mathcal{K}/m \qquad (28)$$

where v_s is the orbital speed of P. Since $\mathcal{K}$ is known, v_s may be determined by (28) and then r_s may be calculated using (11), i.e.

$$r_s = L/mv_s \qquad (29)$$

Finally in this case, it follows by (15) that

$$\mathcal{G} = r_s v_s^2/M \qquad (30)$$

POAMS at the Quantum Level

In POAMS, conservation of angular momentum applies not only on the macro-scale but also on the micro-scale, where the structure of matter is ultimately discrete, with angular momentum quantised in discrete units of $h/2\pi$.

The following example demonstrates that the parameters for the hydrogen atom can be derived purely from considerations of angular momentum alone, without the classical assumption of the existence of an electrostatic force (as in Bohr's original derivation). POAMS conceives the hydrogen atom as an angular momentum system of automatically paired and balanced masses, equivalent to the conventional 'electron' and 'proton'.

Consider once again, a particle *P* of mass *m* orbiting a body *B* of mass *M*, where now the masses are of micro dimensions. Suppose that *P* has a circular orbit about *B* and that $m = 9.1093897 \times 10^{-31}$ kg and $M = 1.6726231 \times 10^{-27}$ kg.

These masses just happen to be the mass of the so-called 'electron' and 'proton' respectively. It must not be supposed that we are implying that the electron physically orbits the proton in a circular orbit, or indeed in any continuous classical orbit.

Assuming that our two-body system has a total angular momentum whose magnitude is $h/2\pi$ and that there are no spin effects present, it follows by (15) and (11) that

$$v_0 = GmM/L = 2\pi GmM/h$$

where v_0 is the orbital speed of *P*. Taking the known values, $h/2\pi \approx 1.054572749 \times 10^{-34}$ Kg m^2 s^{-1} and $G \approx 6.67259 \times 10^{-11}$ N m^2 kg^{-2}, gives

$$v_0 \approx 9.640627 \times 10^{-34} \text{ m s}^{-1}.$$

It then follows by (11) that the radius of the natural circular orbit of *P* about *B* is given by

$$r_0 = L/mv_0 = h/2\pi mv_0 \approx 1.2 \times 10^{29} \text{ m}.$$

This clearly demonstrates that orbital angular momentum alone is not sufficient to explain the parameters of the hydrogen atom. The implication is that, in POAMS, the effects of spin become significant at the quantum level.

Notice that the orbital kinetic energy of the mass *P* in its spin-free natural orbit is

$$K_o = mv_0^2/2 \approx 4.23 \times 10^{-97} \text{ J}.$$

In POAMS, it is necessary to incorporate the intrinsic (spin) kinetic energy of the 'electron' *P* in the calculation of its orbital kinetic energy for its proper orbit about the 'proton' *B*. We proceed by finding the mechanical energy equivalent, *E*, in joules, of the magnitude of the conventional 'electron charge' *e*. This is given by

$$E = I_0 e$$

where I_0 is the kinetic energy required to ionize the hydrogen atom. Here, $I_0 \approx 13.6$ volts and $e \approx 1.60217733 \times 10^{-19}$ coulomb. In POAMS, it is this mechanical energy-equivalent of the static charge which is interpreted as the intrinsic (spin) kinetic energy, K_s, of the 'electron' so that

$$K_s = E \approx 2.179 \times 10^{-18} \text{ J}.$$

The orbital kinetic energy, K_o, of the natural orbit of P, without taking spin effects into consideration, is almost negligible compared to K_s. This means that we can take $\mathcal{K} \approx K_s$, where $\mathcal{K}$ is the orbital kinetic energy of the 'electron' P in its true 'orbit', *independent* of the direction of its theoretical spin in relation to the plane of its theoretical orbit about B.

It then follows from (28) that the orbital speed, v_s, of P in its natural orbit, taking spin effects into account, is

$$v_s = (2\mathcal{K}/m)^{1/2} \approx (2K_s/m)^{1/2} \approx 2.1877 \times 10^6 \text{ ms}^{-1}.$$

Then from (29), the radius, r_s, of the natural orbit of P is given by

$$r_s = (h/2\pi)/mv_s \approx 5.292 \times 10^{-11} \text{ m}.$$

The parameters v_s and r_s are the same as those predicted by Bohr's model and by quantum mechanics for the hydrogen atom. In POAMS, these parameters can be explained in terms of an 'equation of motion' of the form (30) for some particular value of $\mathcal{G}$. For this 'natural orbit' of the 'electron', (30) gives the value of $\mathcal{G}$ as

$$\mathcal{G} = r_s v_s^2/M \approx 1.5142 \times 10^{29} \text{ N m}^2 \text{ kg}^{-2}.$$

In this way, Coulomb's law of electrostatics is replaced with what is virtually the Newtonian gravitational inverse square law, but with a different value of G.

Opened for discussion.