**An Angular Momentum Synthesis of ‘Gravitational’ and ‘Electrostatic’ Forces**

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We believe that in their first flushes of inspiration, what practical physicists such as Newton, Faraday and Coulomb conceived as in vacuo ‘forces’ of ‘gravitation’, ‘magnetism’ and ‘electrostatics’ were not meant to be fixed-for-all-time philosophical statements as to what the world is or how it works. We think that these were no more than convenient working hypotheses as to how to deal practically with certain observational and experimental measures. Accordingly, we suggest that these postulated ‘forces’ of static interaction held responsible for the orbital motions of particles on the micro- as well as the macro-physical levels, may now usefully be replaced by a common factor, angular momentum, which is shown to be sufficient in itself to explain orbital motion. In doing this, we extend and clarify ideas encapsulated in our previous papers concerning angular momentum. This leads us to predict certain measurable physical effects associated with spinning bodies.

**Key words:** conservation, angular momentum, holistic, Newton, gravitation, orbital motion, Mach, normal realism, two-body, spin, kinetic energy, quantum, Bohr, charge, electrodynamic, field-forces.

1. **Introduction**

It is universally acknowledged that Newton was the first to try to explain gravitational effects in a systematic way. Using his three laws of motion, together with his inverse square law for gravitational attraction, he was able to obtain mathematically, amongst other things, the equations for the orbits of the planets. He was also able to deduce all of Kepler’s laws, which had been formulated on the basis purely of observational evidence.

In his first law, Newton declared that all inertial, or force-free motion is naturally rectilinear, which implied that orbital motion is unnatural, caused by an invisible ‘gravitational force’ acting in vacuo. His mechanistic philosophy prevented him from seriously contemplating any holistic or non-local approach to this ‘gravitational’ interaction. For instance, in an oft-quoted passage he states,

“That one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which this action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it.” [1]

However, if a system is supposed to be in a state of overall balance, then that balancing influence cannot be thought of as conducted in Newton’s mechanistic way, as the knock-on effect of one body upon another. A balance can make sense only if conceived holistically. And if that holistic balance is to be maintained, then the motion of each body in the system must be instantly correlated with, or balanced with, that of each and every other, with no question of invisible ‘forces’ being involved.

It is well known that one of the first people to criticize Newton’s mechanical-force theory of gravitation was his contemporary, the Irish philosopher and cleric, George Berkeley. Berkeley argued that to make the vacuum a conductor of theoretical ‘forces’ of any kind was nonsense, and he went so far as to hint that perhaps what was involved might be some direct influence of a universal, holistic kind. Mach developed this argument at the end of the nineteenth century. He formulated a principle, which was, in essence, that the influence of distant matter in the Universe may account physically not only for ‘gravitation’ but also for the local phenomena of inertia, momentum and ‘centrifugal force’.

In the physics-philosophy of Mach, the emphasis was placed on observation, as opposed to theoretical speculation, and such was the influence of Mach’s writings that in physics, from then on, the primacy of physical observations began to be emphasized. It was thus insisted that any model of physical reality should be constructed, as far as possible, using these basically observational terms.

Also, since those pioneering writings of Mach’s, it has become increasingly clear, on experimental grounds, that there is a need for an approach to physics that is both more holistic, or non-local, and more observationally centered. In particular, it has long been evident that holistic laws, such as overall conservation of energy, action, angular momentum, and so on, are at odds with Einstein’s special relativity, in which all physical interaction and information are delayed due to limiting speed c.

We address this need for a holistic directness of physical action by accepting as it stands the empirical evidences of so-called ‘action-at-a-distance’ as simply revealing the natural tendency of objects to move and distribute (balance) themselves in instant conformity with the requirements of the various holistic conservation laws. In this way, we concur with Phipps, Graneau and Assis [2,3,4], that there is an urgent need to rethink...
our ideas of space, time and motion along these more holistic, non-Newtonian lines proposed by Mach.

In particular, our approach to gravitation is to postulate that the natural paths of freely moving bodies are not rectilinear but closed, so that all force-free motion is naturally orbital, with no need to postulate any in vacuo forces being responsible for these curvilinear motions. Our argument, counter to that of Newton, is that it is the holistic conservation of angular momentum that keeps these natural paths in balance.

This holistic approach to gravitation was first proposed in our 1992 Physics Essays paper [5] and then developed and encapsulated in our 1995 Physics Essays paper [6]. As explained in those papers, this alternative approach called for a radically different philosophy of physics from that of Newton, and the one we have adopted is what we have named 'Normal Realism'. In a nutshell, this is an updated approach to physics of the observer-centered, or relationist, sort pioneered by Mach. However, whereas Mach's philosophy emphasized the dependence of science on 'sense-data', Normal Realism emphasizes, pace the later Wittgenstein, the part played by language in the organizing of all sense experience. Thus it rejects the unwanted 'idealistic' element in the positivist or sense-data philosophy of the Vienna Circle, which based itself on Mach and which, in the form of the Copenhagen interpretation, continues to alienate many practically minded physicists. (This is the extreme view, generally regarded as nonsensical, that only what is directly perceived at an instant can be real.)

By dispensing, then, in the essentially Machian way, with all nonsensical talk of light traveling inscrutably in a self-sufficient void, which isolates objects from our perceptions of them, Normal Realism seeks, by methods of linguistic analysis, to remove the philosophical problems created by this traditional two-worlds view of matter and perception. In other words, in the continual linguistic process of ascribing existence to things, in Normal Realism everything begins on a par, basically as an item in the language we evolve to adapt and re-adapt our understanding to our experience of the world. This includes, of course, what we have learned to think of as matter, space, time, gravitation and motion.

In Normal Realism, then, we refer to 'objects', our 'perceptions' or 'ideas' of those objects, the 'spaces' between them and even 'voids' or 'vacua', all in a straightforward, logically evolving ordinary language sense. Also, since perception is not self-supporting (as the 'idealists', or 'solipsists' assumed), we may sensibly refer to a residual class, or 'substratum', of unperceived and perhaps unknown things, without falling foul of the bogus dispute between the positivists ('idealists') and the reactionary antipositivists ('realists'). It is in this ordinary language sense of dealing with things that are 'relative' to perception without being confined to it that we speak of 'space', 'time', 'matter' and 'motion'.

In accordance then, with this Normal Realist philosophy, the substance of the 'Universe' does not consist of bits of pure matter or mass floating about, independently of one another, in a self-sufficient void, in the way Democritus originally envisaged. Nor can there be any question of there being any self-sufficient and isolable measures of mass, length or time. In keeping with our relationist approach, we see the substance of the 'Universe' as an inseparable combination of these measures in the form of angular momentum. That is, we accept the observational evidence as it stands and postulate that all non-interrupted free motion is essentially orbital. In other words, for inertial or 'force-free' motion, all bodies seek to move, if allowed, in closed orbits or trajectories with respect to one another, trajectories of the sort we observe in astronomical space. The reason why freely moving bodies behave the way they do, we argue, is that the angular momentum of the whole system is naturally holistically balanced and conserved.

The natural orbit of any particle may therefore be thought of as the geodesic described by the particle when all restrictions on its motion are removed, that is, when the particle moves freely under the influence of nothing but its own angular momentum or inertia. This method of dispensing with Newton's 'gravitational force' is also employed, of course, in General Relativity. The difference is that in our theory, the natural path of the body is determined by angular momentum considerations alone and not by any underlying space-time geometry.

According to our approach, the weight with which a body presses on the surface of the earth is not the result of some unseen and mysterious 'pull of gravity'. Nor is it, as some imagine on the grounds of the equivalence principle in General Relativity, the effect of the earth's surface somehow accelerating upwards under our feet at 9.8 meters per second per second. We simply assume that the body, where it stands, has insufficient angular momentum to orbit freely (weightlessly), so that, with the angular momentum it has, the force it exerts on a weighing scale is that of the reaction of the earth's surface preventing it from orbiting where it should, much closer to the earth's center. This idea will be made explicit in Example 3.1.

What needs to be stressed before proceeding with this analysis is that it is not our intention to offer a superior way of actually calculating the paths of freely moving macroscopic bodies. At this stage, our suggested philosophical departure from the standard Newtonian account of orbital motion might therefore seem trivial. However, this is not the case. Our holistic approach to angular momentum prompts us to include the effect of spin on orbiting bodies. Consequently, we predict that fast spinning bodies will behave differently from what is classically expected. We shall also demonstrate how our theory applies at the microphysical level, where angular momentum is ultimately quantized in discrete units of $\hbar/2\pi$. Thus, by extrapolating our angular momentum approach down to the quantum level, we indicate the possible redundancy of the traditional field forces not only of Newton, but also of Faraday and Maxwell. In this way, our aim is to further Faraday's attempt to unify the classical conceptions of electric and gravitational fields [7].

It is interesting to note that other recent work, notably that of Kanarev, also emphasizes the central role of conservation of angular momentum, both in the effect of spin on macroscopic orbital trajectories and its quantization in atomic structure [8].
2. Mathematical Preliminaries

Although our theory is based on conservation of angular momentum in a general setting, our mathematical treatment here is necessarily restricted to the simplest case of an isolated system containing two bodies, which illustrates the basic principles.

As in Newtonian physics, the orbital angular momentum \( L \) of a particle \( P \) of mass \( m \), in orbit about a point \( O \), is defined by

\[
L = r \times mv = r \times p
\] (2.1)

where \( r \) is the position vector of \( P \), \( v = dr / dt \) is the velocity of \( P \) and \( p \) is the linear momentum of \( P \), all relative to \( O \), at any instant of time. The orbital angular momentum is thus a vector that lies perpendicular to the plane containing \( r \) and \( p \). By our hypothesis, the orbit of \( P \) is closed. Also according to our thesis, the angular momentum of the particle \( P \) is constant in time, so that \( dL / dt = 0 \). Let \( L = | L | \), the magnitude of \( L \). Since \( L \) is a constant vector, it follows that the orbit of \( P \) lies in a plane and \( L \) is a constant. Since \( dL / dt = 0 \), it follows from (2.1) that

\[
dL / dt = dr / dt \times mv + r \times mdv / dt = v \times mv + r \times ma = r \times ma = 0
\]

where \( a = dv / dt \) is the acceleration of \( P \) relative to \( O \). Since \( m \neq 0 \), \( r \neq 0 \) and \( a \neq 0 \), \( a \) cannot be \( 0 \) since \( v \) cannot be a constant in orbital motion), this implies that \( a \) must be parallel to \( r \) and so, clearly, the acceleration of \( P \) is directed towards \( P \) (2.1).

Since the orbit of \( P \) lies in a plane, it is convenient to use plane polar coordinates \( r \) and \( \theta \), where \( r = | r | \) and \( \theta \) is the angle between the radial vector and some fixed radial axis. The angular speed of \( P \) is then denoted and defined by

\[
\omega = d\theta / dt = \dot{\theta}
\]

where a dot denotes differentiation with respect to \( t \). As in Newtonian theory, it then follows that the orbital speed, \( v = | v | \), of \( P \) is given by

\[
v^2 = \dot{r}^2 + r^2 \omega^2
\] (2.2)

The magnitude of the acceleration, \( a = | a | \), of \( P \) is given by

\[
a = | \ddot{r} - r \omega^2 |
\] (2.3)

and the magnitude of the orbital angular momentum, \( L = | L | \), of \( P \) is given by

\[
L = mr^2 \omega
\] (2.4)

The fact that \( L \) is constant implies that \( r^2 \omega \) is constant.

If, as in the case of Newtonian theory, the acceleration of \( P \) is given by

\[
a = -\alpha \ddot{r} / r^2
\] (2.5)

where \( \ddot{r} = \ddot{r} / r \) and \( \alpha \) is some constant, then a standard derivation shows that if the orbit of \( P \) is closed, then it is an ellipse [9]. Newton explained the fact that \( a \) is given by (2.5) by introducing an inverse square law for his invisible ‘gravitational force of attraction’. However, this, of course, is not the only possible explanation for (2.5). Assuming that the orbit of \( P \) is an ellipse and that \( a = -F(r) \ddot{r} \), for some function \( F \), it follows that \( a \) must take the form given in (2.5). In practice, the orbits of the planets are not exactly ellipses, since there is an observed perihelion shift effect, so that \( a \) cannot be given exactly by (2.5).

In order to illustrate our basic philosophical principles, we shall take a Newtonian-like approach and assume that closed orbits for a two-body system are naturally elliptical. It follows by (2.3) and (2.4) that

\[
amr = | mr\ddot{r} - m\ddot{r}^2 \omega^2 | = | mr\ddot{r} - L\omega | \quad (2.6)
\]

In Newtonian theory, the orbit of the particle \( P \) around a body of mass \( M \) at \( O \) is determined by the presence of a force \( F \) exerted on \( P \), given by

\[
F = ma = -GmM/ \Gamma^3
\]

where \( G \) is the gravitational constant. Hence, although in our approach the orbit is not caused by the presence of such a ‘gravitational force’ \( F \), since we are assuming that the orbit of \( P \) is an ellipse then in addition to (2.5) we may take \( a = GM \). Since \( a \) is directed towards the center of the closed orbit of \( P \), (2.6) then gives

\[
L = amr / \omega + mr\ddot{r} / \omega = GmM / \gamma_0 + mr\ddot{r} / \omega \quad (2.7)
\]

It follows by (2.2) and (2.4) that the kinetic energy, \( K = mv^2 / 2 \), of \( P \) in its orbit is given by

\[
K = m\ddot{r}^2 / 2 + m\ddot{r}^2 \omega^2 / 2 = m\ddot{r}^2 / 2 + L\omega / 2
\] (2.8)

⇒ \( L = 2K / (\omega - m\ddot{r}^2 / \omega) \)

In the case of a circular orbit, the equations given so far simplify. Since \( r \) is a constant in this case, \( dv / dt = 0 \), so that by (2.2) and (2.3)

\[
a = \gamma_0 \omega = v \omega \quad (2.9)
\]

(Since, in general, \( \gamma_0 \) is a constant, it follows that \( v \) and \( a \) are constants in this case.) It then follows from (2.4), (2.7), (2.8) and (2.9) that

\[
L = mvr = GmM / v = 2Kr / v \quad (2.10)
\]

3. A New Approach To Orbital Motion

In declaring inertial motion to be naturally rectilinear, Newton was implying that orbital motion is unnatural, caused by an in vacuo force of ‘gravitational attraction’. He then concluded that orbiting bodies remain separate in the way they do due to the balance of equal but opposite ‘gravitational’ and ‘centrifugal’ forces. For the two-body system described in the previous section, consisting of a mass \( M \) orbiting a mass \( m \), it follows by (2.3) and (2.5) with \( a = GM \) that this balance is expressed by
where \( \omega(t) \) is the angular speed of the mass \( m \) and \( r(t) \) is the distance between \( m \) and \( M \).

Had he been more of an empiricist, Newton might have begun instead with the fact that there are no forces manifest in natural orbital motion. According to our thesis, this is where we should start, with the postulate that all inertial motion is, as it everywhere appears, naturally orbital. In this alternative interpretation, the true (i.e. measurable) force is not concealed in the natural orbit of the particle but is revealed in the difference between that natural orbit and an ‘unnatural’ one, that is, some state of motion or rest in which the particle is held by a measurable force, of magnitude \( F \).

Specifically, for the two-body system (bearing in mind our simplified mathematical approach taken in Section 2), any natural orbit is an ellipse and we have, in particular,

\[
P = |\frac{GmM}{r^2} - GmM| = |G - G'| mM / r^2 \tag{3.2}
\]

where \( G' \) is now the ‘unnatural’ \( G' \) for any ‘unnatural’ (i.e. constrained) elliptical orbit. Otherwise, for a natural force-free orbit, we have \( G' = G \) and \( F = 0 \).

We now demonstrate how our approach works in this specific case. Consider, once again, a particle \( P \) of mass \( m \) orbiting a body of mass \( M \). The magnitude of \( P \)'s orbital angular momentum is given by (2.4). For a particular elliptical orbit, the parameters \( r(t) \) and \( \omega(t) \) are known (they are observational measures) and so \( L \) can be calculated. Suppose that \( P \) is in a constrained elliptical orbit, and let the particle in its natural orbit, for that same orbital angular momentum \( L \), have angular speed \( \omega(0) \) and radial coordinate \( r_0(0) \). Then by (2.7),

\[
L = m\omega_0^2 \omega_0 = \frac{GmM}{r_0} + m\omega_0 r_0 / \omega_0 \tag{3.3}
\]

\[
L = (GmM / r_0 + m\omega_0 r_0 / L) \approx L^2 = Gm^2 M r_0 + m^2 r_0^3 \tag{3.4}
\]

Knowing \( L, G, M, \) and \( m \), this second order differential equation for \( r_0(t) \) can be solved, at least in theory, so that the parameters of the natural orbit of the particle, that is, \( r_0 \) and \( \omega_0 \), can be calculated. The factor \( G' \) which appears in (3.2) for the constrained orbit is then given by (2.7); i.e.,

\[
L = G' m M / r_0 + m r_0 / \omega_0 \tag{3.5}
\]

from which \( G' \) can be calculated. The magnitude \( F \) of the measurable force exerted on the particle in its constrained orbit is then given by (3.2).

Of course, in the ideal case of a circular orbit, this process is considerably simplified. It follows from (2.10) that in this case

\[
L = m\omega_0 r_0 = GmM / v \tag{3.6}
\]

where \( v \) is the orbital speed of \( P \) in its constrained circular orbit of radius \( r \), and

\[
L = m\omega_0 r_0 = GmM / v_0 \tag{3.7}
\]

where \( v_0 \) is the orbital speed of \( P \) in its natural circular orbit of radius \( r_0 \).

### 3.1 Example

Consider the orbital angular momentum relative to the earth's center, of a mass, say, of 1 kg, placed on a weighing-scale at the earth's equator. The speed of the mass \( m \) in this constrained orbit is the revolutionary speed of the earth, namely \( v = 464.74 \text{ m s}^{-1} \). The radius of this orbit is the earth's mean equatorial radius, given by \( r = 6.372808 \times 10^6 \text{ m} \). Since the motion of the mass is circular, the magnitude of its angular momentum is given by (3.6), so that

\[
L = m\omega_0 r_0 = 2.961708085 \times 10^9 \text{ kg m}^2 \text{ s}^{-1}
\]

With that relatively small amount of angular momentum, if we imagine the earth's radius shrunk to the size of a super-dense billiard ball, the natural orbital speed, \( v_0 \), of that kilogram mass, assuming its natural orbit to be circular around the earth's mass, should be given by (3.7), where \( G = 6.67759 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \) and \( M \) is the mass of the earth, so that \( M = 5.976 \times 10^{24} \text{ kg} \). Hence

\[
v_0 = GmM / L = 1.346365 \times 10^5 \text{ m s}^{-1}
\]

Then the radius, \( r_0 \), of this natural orbit, is also given by (3.7), so that

\[
r_0 = L/mv_0 = 2.199781 \times 10^4 \text{ m}
\]

This is approximately \( 1/290 \) of the earth's true radius. Standing still, therefore, near the earth's equator, the mass would seek to orbit at \( 289/290 \text{ m} \) of the distance from the soles of our feet to the earth's center \( \Omega \).

Where it stands then, at the earth's surface, at constrained orbital speed \( v \), the factor \( G' \) for the mass \( m \) in this ‘unnatural’ orbit is given by (3.6), so that \( G' \)

\[
G' = Gv / mM \approx 2.3032534 \times 10^{13} \text{ N m}^2 \text{ kg}^{-2}
\]

The magnitude of the force with which the mass seeks to oppose the intervention of the earth's surface is then given by (3.2), so that

\[
F = m(MG' / r^2) \approx 9.785 \text{ N}
\]

It is this force which produces one kilogram weight on the weighing-scale.

This calculation does not produce exactly the Newtonian result of \( F = 9.80665 \text{ N} \), since the value of \( M \) used is calculated from Newtonian theory and has to be corrected in our thesis, where

\[
Mm(G' - G) / r^2 = mg
\]
Here, in Newtonian terms, \( g \) is the acceleration due to the earth's gravitational pull, so that the value of \( M \) will be larger than the value usually quoted for \( M \). In this case,

\[
M = \frac{g r^2}{(G \cdot G')} = \frac{gr^2}{(G \cdot v^2r / M)}
\]

\[
\Rightarrow gr^2 - MG - rv^2 = M = r(gv^2 / G)
\]

Hence \( M = 5.989 \times 10^{24} \text{ kg} \) and the force exerted on one kilogram weight is given by

\[
F = GmM(G \cdot G') / r^2 = 9.81 \text{ N}
\]

Since, in this example, \( v_0 \), \( r_0 \) and \( G' \) do not depend on the given mass \( m \), it follows that in general, the magnitude of the force with which any mass \( m \) kg seeks to oppose the intervention of the earth's surface is given by

\[
F = mGm(G \cdot G') / r^2 = 9.81 \text{ mN}
\]

(3.8)

3.2 Example

Consider a car of mass \( m = 1500 \text{ kg} \), say, resting somewhere near the earth's equator. Then from (3.8), the car will press on the road's surface, with a force, of magnitude given approximately by

\[
F = 9.81 \times 1500 = 14715 \text{ N}
\]

and so weighs 1500 kg.

Now suppose the car follows the sun by driving west along some equatorial highway at a speed of 464.74 m s\(^{-1}\), so that it counteracts the earth's rotational speed. Hence, in effect, the car is stationary relative to the revolving earth, and so its constrained orbital speed, relative to the earth's center, is \( v = 0 \). It follows that the magnitude of its orbital angular momentum is given by \( L = 0 \). In this degenerate case, according to the argument used in Example 3.1, \( r_0 = 0 \). (Since \( L = 0 \), the car would seek the center of mass of the orbit.) Then \( G' = 0 \) and so the car would press on the road's surface, which is scudding eastwards under its wheels at 464.74 m s\(^{-1}\), with a force given by

\[
F = GmM / r^2 = 14760 \text{ N}
\]

(3.8) and the spin angular momentum \( s \) as:

\[
J = L + S
\]

(4.1)

Note that when there is no orbital motion, \( J = S \). Thus \( s \) can be thought of as the intrinsic angular momentum of a particle.

Consider once again, the two-body case consisting of a particle \( P \) of mass \( m \) orbiting a body of mass \( M \). According to Newtonian dynamics as applied to rigid bodies, if the particle is spinning, the inverse square law has to be applied to each ‘elementary piece’ of the rotating particle, in dealing with which we are faced with the well-known many-body problem which is notoriously intractable. It can be shown that if the spinning body is perfectly spherical, in applying Newton's laws it acts like a point mass, so that the spin situation is the same as that of the non-spin. In other words, in Newtonian dynamics, the same orbit is predicted in both cases.

The main difference, therefore, between our approach and Newton's is that since we are supposing that in a system that is holistically balanced, any orbital angular momentum is balanced against the total momentum \( J \) of the system, introducing a spin at any one place has a direct and immediate effect on the system throughout. Such effects are not normally observable. They are nevertheless implicit, as may now be seen. Let \( J = |J| \), \( L = |L| \) and \( S = |S| \). It then follows from (4.1) that if \( s \) points in the same direction as \( L \), (so that the particle spins in the same plane and in the same direction as its orbit), then

\[
J = L + S
\]

(4.2)

On the other hand, if \( s \) points in the opposite direction to \( L \), (so that the particle spins in the same plane as its orbit but in the opposite direction), then

\[
J = |L - S|
\]

(4.3)

(In general, \( J \) is given by \( J^2 = L^2 + S^2 + 2LS \cos \varphi \) where \( \varphi \) is the angle between \( L \) and \( S \).)

When investigating these spin effects it is useful to consider orbital and spin kinetic energies. Recall, from Section 2, that the kinetic energy, \( K \), of the orbiting particle in the two-body situation is given by (2.8). Take the case of a particle \( P \) of mass \( m \) orbiting a body of mass \( M \). If \( P \) is not spinning and is following its natural elliptical orbit with angular coordinate \( \eta_0(t) \); and if it has an angular speed \( \omega_0(t) \), then from (4.1) and (3.3),

4. Angular Momentum And Spin
\[ J = L = mr_0^2 2ω_0 = GM / r_0ω_0 + mr_0^2 / ω_0 \] (4.4)

From (2.8), the orbital kinetic energy \( K_o \) of that natural orbit is then given by

\[ K_o = \frac{m r_0^2}{2 + L^2 / 2mr_0^2} \] (4.5)

Now suppose that the particle, whilst orbiting, is also spinning and that its spin kinetic energy is \( K_s \). The magnitude, \( S \), of its spin angular momentum is given by

\[ S = 2K_s r / v \] (4.6)

where \( r \) is the spin radius and \( v \) is the rotational speed (4). Compare this equation with (2.10). Suppose also that the particle spins in the same direction and in the same plane as its orbit. Let \( r^s(t) \) be the radial coordinate for that orbit when the effect of the spin is taken into account, and let \( ω^s(t) \) be the angular speed of the particle in that new orbit. At this point, our holistic approach requires a departure from standard Newtonian mechanics. It is that, by analogy with (2.8), (2.4) and (4.2), we postulate that

\[ L = 2K / ω^s \cdot m(r^s)^2 / ω^s = m(r^s)^2 / ω^s \]

\[ ⇒ L^2 = 2Km(r^s)^2 - m^2 (r^s ω^s)^2 \] (4.7)

where \( L \) is the magnitude of the orbital angular momentum and

\[ K = K_o + K_s \] (4.8)

Equation (4.7) is a first order differential equation for \( r'(t) \) and so, at least in theory, \( r(t) \) and hence \( ω(t) \) can be found.

Spin effects can be taken into account by adapting (2.7), replacing \( a \) by a function \( G \) which depends on the parameters \( r^s \) and \( v^s \). It then follows by (2.7) that \( G \) is obtained by solving

\[ L = G m M / r^s ω^s + mr^s ω^s / ω^s \] (4.9)

Note that \( G \rightarrow G \) as \( K_s \rightarrow 0 \).

By analogy with (4.3), by the same token, we postulate that in the situation where the particle spins in the opposite direction and in the same plane as its orbit, the above equations still hold, except that in this case, (4.8) is replaced by

\[ K = |K_o - K_s| \] (4.10)

As in Section 3, for circular orbits this process is considerably simplified. Equation (4.4), for no spin, becomes

\[ J = L = mv_0^2 / 2 = GmM / v_0 \] (4.11)

where \( v_0 \) is the speed of the particle in its natural orbit. In this case

\[ K_o = mv_0^2 / 2 \] (4.12)

and (4.7) becomes

\[ L = 2K r^s / v^s = m v^s r^s \Rightarrow (v^s)^2 = 2K / m \] (4.13)

which is easily solved to obtain the orbital parameters \( v^s \) and \( r^s \). Finally, (4.9) becomes

\[ L = GmM / v^s \] (4.14)

from which the new ‘gravitational’ factor \( G \) is calculated [10]. A major difference then, between our angular momentum account of orbital motion and Newton’s ‘gravitational’ account, is that in ours, the spins of bodies affect their orbital parameters in a way that cannot be accounted for by purely ‘gravitational’ mass attraction.

### 4.1 Example

Consider a spinning disc of weight 175 gm, as in experiments performed by Hayasaka and his coworkers [11,12]. In contrast to these experiments, we shall assume that the disc spins at the equator and in the same plane as that of the earth's rotation. As an inert mass, this disc would orbit the earth in accordance with (4.11), with a natural orbital angular momentum

\[ L = 5.1829914826 \times 10^8 \text{ kg m}^2 \text{ s}^{-1} \]

and a natural orbital speed of

\[ v_0 = GmM / L = 1.349293731 \times 10^5 \text{ m s}^{-1} \]

The kinetic energy of this natural orbit is then given by \( K_o = m v_0^2 / 2 \), where \( m = 0.175 \text{ kg} \), so that

\[ K_o \approx 1.59301937595141 \times 10^9 \text{ Kg m}^2 \text{ s}^{-2} \]

Suppose that, as in the Hayasaka experiments, the disc spins at 18000 rpm and has a moment of inertia given by

\[ I \approx 969.47 \text{ gm cm}^2 = 9.6947 \times 10^5 \text{ kg m}^2 \text{ s}^{-2} \]

Then (4) its spin kinetic energy is given by

\[ K_s = Iω^2 / 2 = (600π)^2 / 2 \times 9.6947 \times 10^5 \]

\[ = 1.72229130941 \times 10^6 \text{ Kg m}^2 \text{ s}^{-2} \]

Then from (4.13), if the disc spins in the same direction and in the same plane as its orbit around the earth, it natural orbital speed is given by

\[ v^s = \sqrt{2(Ko + K_s) / m} = 1.349293804 \times 10^5 \text{ m s}^{-1} \]

It then follows by (4.11) and (4.14) that the ‘gravitational factor’ \( G \) for this new orbit is given by

\[ GmM / v^s = G = v^s G / v_0 = 1.000000054 \]
In this sense, therefore, the spinning disc is more 'strongly attracted' towards the earth, then if it were not spinning. If the disc spins in the direction opposite to that of its orbital motion, then the 'gravitational factor' is given by

$$G = \frac{v^*}{v_0}$$

where

$$v^* = \sqrt{2\left(K_0 - K_s\right)/m}$$

so that

$$G = 0.999999962G$$

In this case, the spinning disc is, in a sense, 'repelled' to some extent by its spin.

It follows by Example 3.1 that the weight of the disc, when not spinning, is given by the weight equivalent of the force with magnitude

$$F = mM(G\cdot G')/r^2 \quad \text{where} \quad G' = L_0/mM$$

In this case,

$$F \approx 0.175 \times 9.805865398 \text{N} = 0.175 \text{kg}$$

If the disc is spinning in the same direction as its orbital motion, its weight is given by a 'force equivalent' of

$$F = mM(1.000000054G\cdot G')/r^2 \approx 0.175 \times 9.8058659298 \text{N}$$

$$\Rightarrow F \approx 0.17500000945 \text{kg}$$

In other words, the spinning disc is about one hundredth of a milligram heavier. If the disc spins in the direction opposite to that of the orbital motion, it weighs about one hundredth of a milligram less. The same changes occur, of course, to the 'acceleration of the object due to gravity'. By above, if the disc is not spun and is dropped towards the surface of the earth, its acceleration is about 9.805865398 m s\(^{-2}\), whereas if it is spun in the same direction as its orbital motion and is then dropped, its acceleration is about

$$9.8058659298 \approx 1.000000054 \times 9.805865398 \text{m s}^{-2}$$

It then follows by (4.11) and (4.14) that G for this new orbit is

$$G \approx 1.000004G$$

If the disc spins in the opposite direction, then

$$G \approx 0.999996G$$

It follows by Example 3.1 that if the disc is spinning in the same direction as its orbital motion, then its weight is given by the 'force equivalent'

$$F = mM(G\cdot G')/r^2 \approx 0.175 \times 9.8058659298 \text{N}$$

$$\Rightarrow F \approx 1.000004 \times 2.5 \text{kg} \approx 2500.01 \text{gm}$$

In other words, the weight of the ball is increased by approximately one hundredth of a gram. Whereas, spinning in the opposite direction, the ball weighs one hundredth of a gram less.

5. Angular Momentum at the Quantum Level

This departure of ours from Newtonian dynamics for macroscopic bodies might seem unnecessarily radical for physics at that level but this conceptual change is fundamental to our attempt at unification of classically conceived 'forces' on the micro level. For it must be remembered that in our Normal Realist, holistic approach, conservation of angular momentum applies not only on the macro scale but also on the micro scale. We shall show that in contrast to the macro level, on the micro level the intrinsic angular momentum (commonly called 'spin') predominates over the orbital angular momentum. This accords well with the fact that those elementary particles conventionally called "electrically charged" are much more volatile in terms of the strengths of the classically conceived forces they exert on one
another compared to those of ‘gravity’. Indeed, so spectacularly
different are the strengths of these forces that they have been
imputed to ‘powers’ that differ not only quantitatively but also
qualitatively from that of ‘gravity’. This, we maintain, is why
these powers have been allocated special units such as, for
instance, ‘coulombs’ for ‘electric charge’. All these other forces,
however, are conceived by analogy with ‘gravity’, by the
introduction of an inverse square law, such as Coulomb’s law of
electrostatics. However, essentially, no attempt has ever been
made to explain what these esoteric ‘powers’ are. They are
defined by nothing more than their ability to influence the
motions of objects and are therefore ultimately cashable in
mechanical units of joules. Our ultimate thesis is that these
different-strength forces can be unified by expressing them all
dynamically, in the same basic units of kilograms, meters and
seconds.

The following example shows that the parameters for the
hydrogen atom can be derived in this way, purely from
considerations of angular momentum, without the classical
assumption of the existence of an electrostatic force (6). The
Balmer formula for the spectral lines associated with hydrogen
was established by trial and error. As is well known, Bohr’s
model of the hydrogen atom supplied an explanation of the
Balmer formula, on the basis of the electro dynamical theories of
Faraday and Maxwell, with the ad-hoc addition of the Planck
relation to provide the essential discreteness of the possible
associated energies [14]. We approach the explanation of the
parameters of the hydrogen atom differently, by replacing the
conventional ‘Coulomb force’ with angular momentum
considerations. We think of the hydrogen atom as an angular
momentum system of automatically paired and balanced masses,
equivalent to the conventional ‘electron’ and ‘proton’. (The fact
that we do not require quantum mechanics can be justified by
Ehrenfest’s theorem, which states that the expectation values of
quantum mechanical operators behave in the same manner as do
the corresponding systems in classical mechanics [15]. Also, the
speeds of ‘electrons’ in Bohr’s ‘orbits’ are of the order 0.01c, so
that atomic systems can be adequately described by Newtonian-
like dynamics.)

5.1 Example

Consider once again, the two-body situation, in which now
the masses of the particles involved are of micro dimensions. For
simplicity, consider a mass of 9.1093897 x 10^{-31} kg in a circular
orbit around a central mass of 1.6726231 x 10^{-27} kg. These, let
us say, just happen to be the mass of the so-called electron and
proton respectively, in the Bohr hydrogen atom. It must not be
supposed that we are implying that the electron physically orbits
the proton in a circular orbit, or indeed in any continuous
classical orbit. Hence we are not attempting to resurrect the Bohr
model of the hydrogen atom. According to our philosophy,
at the ultimate microphysical level, the paths of the elementary
particles are as discrete as the particles themselves, with angular
momentum that is quantized in amounts irreducible below an
absolute limit of \( \hbar / 2\pi \). We emphasize that our approach is not
simply a mathematically convenient description of ‘the atom’, as
in quantum mechanics. It is also, and essentially, philosophically
consistent with our stated Normal Realist standpoint. The fact,
however, that our model is not intended specifically as a physical
working model does not prevent us from associating vectors
with the quantities involved, in the same way that vectors can be
associated with the operators of quantum theory [16].

Assuming then that our two-body system has a total angular
momentum whose magnitude is \( \hbar / 2\pi \) and that there are no spin
effects present, it follows by (3.7) that

\[
L = \frac{\hbar}{2\pi} = mv_0r_0 = \frac{GmM}{v_0}
\]  

(5.1)

where \( r_0 \) is the radius of the natural orbit of the mass \( m \)
around \( M \), and \( v_0 \) is its orbital speed. Taking the known values,
\( \hbar / 2\pi \approx 1.054572749 \times 10^{-34} \text{ Kg m}^2 \text{s}^{-1} \) and \( G = 6.67259 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \),
(5.1) gives

\[
V_0 = \frac{GmM}{(\hbar / 2\pi)} \approx 9.640627 \times 10^{-34} \text{ m s}^{-1}
\]

and

\[
r_0 = \frac{(\hbar / 2\pi)}{mv_0} = 1.2 \times 10^{-9} \text{ m}
\]

Hence, in this scenario, the ‘electron’ orbits the ‘proton’ at a truly
distinct, distant distance, at almost zero speed! This is clearly
nonsense, and demonstrates that orbital angular momentum
alone is not sufficient to explain the parameters of the Bohr atom.
The implication, in our philosophy, is that the effects of spin
become significant at the quantum level.

The hypothesis that the ‘electron’, considered as an
elementary particle, has an intrinsic angular momentum, as
though it were spinning, was first introduced by Uhlenbeck and
Goudsmit in 1926 [17]. This, of course, was not known to Bohr
when he proposed his model for the atom in 1913. According to
our approach, it is the presence of this spin angular momentum
which provides the correct parameters for the hydrogen atom,
without assuming the existence of an electrostatic force.

In keeping with our hypothesis of angular momentum
conservation, we need to incorporate this spin angular momentum of the ‘electron’ (our mass \( m \)) with its orbital angular
momentum within a single system. This means that in
accordance with our technique introduced in Section 4, we find
the energy-equivalent, \( E \), in joules, of the magnitude of the
conventional ‘electron charge’, \( e \). This is

\[
E = \frac{\hbar cR}{I_0 e}
\]  

(5.2)

where \( \hbar \) is Planck’s constant, \( c \) is the usual electromagnetic
constant, \( R \) is Rydberg’s constant and \( I_0 \) is the ionization
potential for hydrogen, i.e. the kinetic energy required to ionize
the hydrogen atom [14]. Here, \( I_0 = 13.6 \text{ volts and } e = 1.60217733 \times 10^{-19} \text{ coulomb} \). In our approach, it is this mechanical energy-
equivalent of the static charge which we interpret as the intrinsic
(spin) kinetic energy \( K_s \) of the electron, so that by (5.2), it
follows in this case that

\[
K_s = E = I_0 e = 2.179 \times 10^{-18} \text{ J}
\]
Notice that the purely orbital kinetic energy of the natural orbit of the mass \( m \), i.e., without taking spin effects into consideration, is, by above

\[
K_o = 4.23 \times 10^{-7} \text{ J}
\]

and so is almost negligible compared to \( K_{g^*} \). This means that we can take \( K = K_{g^*} \), in the notation of Section 4, independently of the direction of the theoretical spin of the mass \( m \) in relation to the plane of the theoretical orbit of \( m \) about \( M \). (7)

It then follows from (4.13) that the speed \( v^* \) of the mass \( m \) in its natural orbit, taking spin effects into account, is

\[
v^* = \frac{\sqrt{2K/m}}{m} = 2.1877 \times 10^6 \text{ m s}^{-1}
\]

Thus from (4.13), the radius \( r^* \) of the natural orbit of the mass \( m \) is

\[
r^* = \frac{\hbar}{2\pi m} \approx 5.292 \times 10^{-11} \text{ m}
\]

These parameters \( v^* \) and \( r^* \) are the same as those predicted by Bohr's model [14] and by quantum mechanics for the hydrogen atom [18].

Continuing with the approach of Section 4, for this 'natural orbit', (4.14) gives the 'gravitational factor' \( G \) as

\[
G = \frac{L v^*}{m L} = \frac{\hbar v^*}{2\pi m M} \approx 1.5142 \times 10^2 \text{ N m}^2 \text{ kg}^{-2}.
\]

In this way, Coulomb's law of electrostatics is replaced with what is virtually the Newtonian gravitational inverse square law, but with a different value of \( G \). The reason, of course, for this huge increase in the value of \( G \) is the presence of the relatively enormous amount of spin kinetic energy of the mass \( m \).

It needs to be stressed that our aim in this example is to demonstrate philosophically how those orbits Bohr calculated in terms of a Newtonian dynamics laced with the electrodynamics of Faraday, Maxwell, Coulomb and others, could logically have been derived from Newtonian-type dynamics alone, simply by altering the value of the 'gravitational constant' \( G \). And again it must be emphasized that what we are presenting here is not some new concept of the atom to compete with the current physical models that superseded Bohr's. That is to say, it is by no means an exercise in up-to-date physics. It is essentially a logical demonstration of how, ahistorically, our concepts of motion and distant interaction might have developed in direct observational terms, without postulating invisible intermediaries such as 'gravitational' and 'electrostatic' forces, linking one atom or part of an atom to another.

We can generalize the above argument by considering the case of an electron in what is conventionally called an 'excited state' in the hydrogen atom. Consider, once again, the case of a particle of mass \( m \) orbiting a particle of mass \( M \) in a circular orbit, where \( m \) is the mass of the 'electron' and \( M \) is the mass of the 'proton'. Suppose that the magnitude of the angular momentum of the system is \( n\hbar/2\pi \) where \( n \) is a positive integer greater than 1. Suppose first of all, that there are no spin effects present. Let \( v_{0n} \) denote the speed of the non-spinning mass \( m \) in its natural orbit and \( r_{0n} \) the radius of that orbit. Let \( v_0 = v_0 \) and \( n_0 = r_0 \).

It follows by (5.1) that

\[
v_{0n} = \frac{GM}{n(\hbar/2p)} = v_0/n \quad \text{and} \quad n_{0n} = n(n/2\pi) / m v_{0n} = n^2 r_0
\]

and, once again, these parameters are clearly nonsense. Note that the orbital kinetic energy in this case is given by

\[
K_{on} = m v_{0n}^2 /2 = \frac{K_o}{n^2} \quad \text{where} \quad K_o = K_{01}
\]

But now if, instead, we associate a spin kinetic energy, \( K_{sn} \), equal to the energy level of the orbit, we obtain

\[
K_{sn} = \hbar c R / n^2 = \frac{K_o}{n^2} \quad \text{where} \quad K_o = K_{s1}
\]

Then, for any value of \( n \), \( K_{sn} >> K_{on} \) since \( K_o >> K_o \). From this it follows, by (4.13), that the parameters, \( v^*_n \) and \( r^*_n \) for the 'corrected' natural orbit are given by

\[
(v^*_n)^2 = 2K_{sn} / m = 2K/n^2 m \Rightarrow v^*_n = \frac{v^*_n}{n} \quad \text{where} \quad v^*_n = v^*_1
\]

and

\[
r^*_n = n(n/2\pi) / m v^*_n = n2r^*_n \quad \text{where} \quad r^*_n = r^*_1
\]

so that the correct allowable orbits, as derived in both Bohr's model [14] and quantum theory [18] are deduced.

The most important factor, then, in our account of the hydrogen atom is that the conventional 'charge' associated with an electron is interpreted as its intrinsic angular momentum. Notice that our approach assumes an explicit interaction between the orbital and spin angular momentum of the electron, exactly as required by quantum theory, supported by electromagnetic theory [8].

5.2 'Attraction' and 'Repulsion'

In the standard electrodynamic approach, the behavior of an 'electron' in relation to a 'proton' is explained by ascribing to each particle a purely static property called 'charge', of equal magnitude but of opposite sign. The resulting 'attractive force' between the two particles, by Coulomb's law, countered by 'centrifugal force', is thus thought to determine the orbit of the one around the other by analogy with Newton's account of our earth's orbit around the sun. In contrast, our hypothesis, which is virtually a tautology, is that all orbital motion is force-free and that the 'electron' orbits the 'proton' in the way it does simply because, by definition, this is what that amount of angular momentum is.

In our Normal Realist approach, therefore, all talk of those 'attractive forces' determining the orbits, of planets around stars, electrons around protons and so on, becomes redundant. By that same token, there can be no static repulsion as such, between particles but only degrees of what Newton would have regarded as 'attraction'. In that Newtonian sense, a particle orbiting a central particle 0 at a mean radius \( r_1 \) is 'more strongly attracted' to 0 than another particle orbiting at mean radius \( r_2 > r_1 \). In this way, the results of the previous example imply that the static Newtonian 'attraction' of a mass \( m \) towards 0, in orbiting that
central particle, is determined simply by the magnitude of the dynamic spin angular momentum intrinsic to the mass \( m \) - the greater the spin angular momentum of \( m \), the higher the degree of its Newtonian 'attraction' towards \( 0 \), as long as its spin and orbital angular momentum vectors remain parallel. In general, it can easily be deduced from the results of Section 4 that the addition of spin angular momentum to the mass \( m \) alters its orbit in relation to \( 0 \), either towards or away from \( 0 \), depending on the chosen direction of the spin of \( m \). Hence, as we have seen, in our thesis, spin affects the relative Newtonian 'attraction' of \( m \) to 0 in a way that it cannot according to Newton's thesis, where the addition of spin angular momentum does not change the sign of the uniformly 'attractive' 'centripetal force' \( F \).

We claim, then, that this Newtonian, statically conceived 'attraction' of the electron to the proton is so much larger than that supplied by 'gravity' because of the enormous amount of intrinsic spin angular momentum associated with the electron. If the total spin angular momentum were reduced to zero, then the behavior of the electron with respect to the proton would revert to that described by the first part of the preceding example, where the degree of 'attraction' between the two particles is minimal. This would be tantamount to the sort of 'repulsion' and complete ionization that is described in classical electrodynamics.

We further our discussion of 'attraction' and 'repulsion' by considering the apportioning of a spin angular momentum to each of the conventional masses in the hydrogen atom. As in quantum theory, we associate an intrinsic angular momentum of magnitude \( h/4\pi \) with the electron, assuming, conventionally, that it behaves as if its spin angular momentum vector is perpendicular to the plane of its orbit \( \theta \). Since the magnitude of the total angular momentum for the 'ground state' orbit of the electron is \( h/2\pi \), it is logical, in our approach, to consider the hydrogen atom as if it were a two-body system consisting of a spinning mass \( m \) orbiting another spinning mass \( m \), in which both masses have an associated spin angular momentum vector, the magnitude of the sum of these vectors being \( h/2\pi \).

For simplicity, in view of the canceling-out of other possible angular momentum effects, we may suppose that the total angular momentum is derived from the spin angular momentum of both masses, each with magnitude \( h/4\pi \). Since the magnitude of the total angular momentum is \( h/2\pi \), the spin angular momentum vectors of the masses must be considered as pointing in the same direction. Theoretically, if the two vectors were pointing in opposite directions, then the magnitude of the total angular momentum would be zero, so that the whole atom would collapse. In this sense, equal angular momentum vectors give rise to a 'repulsive' effect, without the presence of which the atomic system could not exist. In other words, 'like spins repel', and 'unlike spins attract', just as, in the standard picture, two spinning electrons with like poles repel and two spinning electrons with unlike poles attract. This conclusion may be compared with what we have said previously. The statement in classical electrodynamics that 'unlike charges attract' has been replaced in our approach by concluding that the magnitude of the added spin angular momentum of the electron gives rise to a relative 'attraction' between the electron and the proton in the form of the classical Bohr atom. This classical atom does not collapse completely since there is also a 'repulsive' effect between the electron and the proton in the sense that their associated spins are alike. We conclude that it is this balance between these 'attractive' and 'repulsive' effects, not that of static 'charges' in centrifugal motion, that provides the true picture.

6. Conclusion

If our arguments are correct, then there is no necessity for thinking of bodies being attracted and repelled by any sorts of invisible in vacuo field forces, either by 'gravitational forces' on the macro scale or 'electrostatic forces' on the micro scale. From our Normal Realist standpoint, these forces and force fields are fictitious. Our ultimate thesis is that with appropriate step-changes in the value of our generalized 'gravitational factor' \( G \), the parameters of all particle orbits may be explained in terms of the various involutions and convolutions of angular momenta. This suggests one possible method of solving the notorious 'unified field' problem - that is, by dispensing with those mystifying 'fields' and associated 'forces' altogether - a unified non-field theory, as it might be called [19].

Insofar as the angular momentum nexus may be regarded as a 'field', it is of the instantaneous 'action-at-a-distance' sort envisaged, in electromagnetic contexts, by Weber and Helmholtz, rather than the time-delayed 'propagated' kind postulated by Faraday and Maxwell [20]. In our theory, the propagation customarily called 'electromagnetic' consists of a time-delayed movie-like sequence of instantaneous quantum states of the angular momentum field at the rate \( c \) of length-units to time-units customarily called 'the speed of light' [21,22].

In summary, then, we maintain that it is the holistic requirement for angular momentum conservation which makes every freely moving bit of matter move in proper-time-instantaneous response to each and every other, in the way Phipps, Graneau, Assis and others have suggested along the lines envisaged by Mach.

Endnotes

{1} This is in contrast to Newtonian physics, which postulates that the orbit of a central force \( F \) is due to the existence of a central force \( F \) given by \( F = ma \cdot \hat{F}(r)r \) and then \( dL/dt = r \times ma = -F(r)(r \times r) = 0 \) so that \( L \) is a constant.

{2} Note that the parameters \( v_0 \) and \( t_0 \) of the natural orbit do not depend on the actual mass \( m \) since \( v_0 = \frac{GM}{L} = \frac{GM}{vr} \) and \( t_0 = L/mv_0 = vr/v_0 = (vr)^2/GM \).

{3} Note that \( G \) does not depend on the given mass, since \( G = Lv/mM = v^2r/M \).

{4} If the particle has angular speed \( \omega \) and moment of inertia \( I \), then \( S = I \omega^2 \). (For example, for a sphere of uniform density, \( I = 2mr^2/5 \), where \( m \) is the mass of the sphere and \( r \) is its radius.) The spin kinetic energy \( K_s \) of the spinning particle is defined by \( K_s = I \omega^2 / 2 \), so that, since \( v = \omega r \), \( S = 2K_s/\omega = 2K_s r/v \)
{5} We are indebted to Dr. Colin Evans of the Department of Physics, University of Wales, Swansea, for this information.

{6} The hydrogen atom constitutes the simplest atomic system and as such, it has played a crucial role in the history of modern physics as a testing ground for atomic theories.

{7} According to quantum mechanics, there are only two possible spin states for the electron, conventionally ‘up’ or ‘down’. In our approach, it is possible conventionally to assign a spin angular momentum to the ‘electron’, which points either parallel to the orbital angular momentum or in the opposite direction. Since, from Section 4, \( K = K_o + K_s \) in the first case and \( K = K_o - K_s \) in the second, \( K \approx K_s \) in either case, so that the direction of spin has no effect on the parameters of the orbit of the mass \( m \).

{8} According to classical electrodynamics, an orbiting electric charge produces a magnetic field, so that the electron generates a magnetic moment proportional to its orbital angular momentum. Similarly, the spinning electron possesses an intrinsic magnetic moment proportional to its spin angular momentum. This interacts with the magnetic moment proportional to its orbital angular momentum, so that there is an implicit interaction between the orbital and intrinsic angular momentum of the electron.

{9} In quantum mechanics, electron spin was originally introduced to explain empirical evidence concerning atoms with a larger atomic number than hydrogen. For example, the observed ionization energy of mercury is partially explained by a larger atomic number than hydrogen. For example, the observed ionization energy of mercury is partially explained by an atomic number greater than hydrogen. For example, the observed ionization energy of mercury is partially explained by an atomic number greater than hydrogen.

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